BETTING ON WHAT IS NEITHER VERIFIABLE NOR FALSIFIABLE

You might be willing to bet your life on it, but how?







VERIFIABLE, FALSIFIABLE

- Δ_0 : Questions whose answers we will find out at a definite point in time.
- Verifiable: Statements that will be confirmed (if true)
 - "There is at least one white swan"
 - "An asteroid will hit the Earth"
 - "You are mortal"
 - "P halts"
- Falsifiable: Statements that will be refuted (if false)
 - "All swans are white"
 - "An asteroid will never hit the Earth"
 - "You are immortal"
 - "P doesn't halt"

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- "All men are mortal"
- "Every swan will grow a white feather"
- "This infinite checkerboard has a completely shaded horizontal row"
- "There are an infinite number of primes"
- "The limit of this sequence is L"

• This feels like it should be meaningful!





ARITHMETICAL HIERARCHY

An obvious, immediate solution might look something like an inductive construction, where for $P \in \Pi_n$ sentence, you can give a trader the asset $\bigvee_{x \leq n} P(x)$ on day n, taking it back the next day, ad infinitum, so the asset value approaches the \sum_{n+1} asset $\exists x, P(x)$ over infinite time.



Theorem 0 (the problem is hard). Let $P(x_1, \ldots x_n)$ be an *n*-ary primitive relation (denote its Σ_n and Π_n quantifications as ΣP and ΠP respectively), let $D: \mathbb{N} \to \text{list } \mathbb{N}^n$ be some fixed "enumerator" of \mathbb{N}^n (i.e. $\text{set}(D_{t+1}) \supseteq \text{set}(D_t)$, $\bigcup_{t \in \mathbb{N}} \text{set}(D_t) = \mathbb{N}^n$), and allow vectorizing P on finset \mathbb{N}^n , i.e. $P(\langle \mathbf{x}_1, \ldots \mathbf{x}_n \rangle) = \langle P(\mathbf{x}_1), \ldots P(\mathbf{x}_n) \rangle$. A "mechanism" for ΣP (respectively ΠP) can be either:

- [asset] a computable sequence of computable functions $v_t : \operatorname{Bool}^{D_t} \to \mathbb{R}$ such that $\lim_{t\to\infty} v_t(P(D_t)) = \begin{cases} \$1 & \text{if } \Sigma P \\ \$0 & \text{if } \neg \Sigma P \end{cases}$ (respectively ΠP).
- [score] a computable sequence of computable functions $s_t : (0,1) \to \text{Bool}^{D_t} \to \mathbb{R}$ such that $\lim_{t \to \infty} s_t(p, P(D_t)) = \begin{cases} \log(p) & \text{if } \Sigma P \\ \log(1-p) & \text{if } \neg \Sigma P \end{cases}$ (respectively ΠP).

Then for n > 3, there is no computable procedure that, given some *n*-ary *P*, gives a mechanism for ΣP (respectively ΠP).

Proof. Suppose such a computable procedure existed, denote it by ϕ . Then for all *n*-ary primitive relations *P*, the sentence ΣP would be equivalent to either $\lim_{t\to\infty} \phi(P)_t(P(D_t)) = 1$ or $\lim_{t\to\infty} \phi(P)_t(0.3, P(D_t)) = \log(0.3)$, depending on the type of mechanism. However, both of these sentences are Π_3 , and for every Σ_n sentence were equivalent to a Π_3 one would contradict Tarski's theorem.

AAAAHHH

LET'S PLAY A GAME.

GAME SEMANTICS

FBAFAKJDJFNSF

- Wait
- How about this game
- " $\forall x, \exists (y,z), BB(z) = y \land y > x$ "

CONSTRUCTIVISM

The Probability that WE WILL
CONSTRUCT an x such that for all y
WE WILL CONSTRUCT, P(x, y)

SOFT NON-CONSTRUCTIVISM

An event *I* that gives no information on *A* or on *B*, but gives information on $A \cup B$.

I have two children; *A* is "my eldest is a boy"; *B* is "my youngest is a boy"; *I* is "I have a boy and a girl".

HARD NON-CONSTRUCTIVISM

An event *I* that is independent of every finite union $\bigcup_{i \in S} A_i$ but informs the countable union $\bigcup_{i \in \mathbb{N}} A_i$

(Not really possible in probability theory)

PROBABILISTIC CONSTRUCTIVISM

Definition 1 (Probabilistic constructivism). Define $\Omega = \{0, 1\}^N$, i.e. the set of binary sequences, to be our "sample space". We define our "constructible algebra" Ψ on this space as the smallest set such that:

- every subset $\overline{\pi}_i = \{ w \in \Omega \mid w_i = 0 \}$ and $\pi_i = \{ w \in \Omega \mid w_i = 1 \}$ is in Ψ .
- The finite unions and intersections of π_i, π_i are in Ψ; these are called its "propositional sets"
- Ψ is closed under unions and intersections over the its computably enumerable subsets, i.e. for $p : \mathbb{N} \to \Psi$ computable, $\bigcup_{x \in \mathbb{N}} p(x)$ and $\bigcap_{x \in \mathbb{N}} p(x)$ are in Ψ .

(*Notes:* This differs from "monadic algebras", "quantifier algebras" and "polyadic algebras" but is good enough for us.)

Then a "constructivist probability assignment" \mathbf{Pr} on Ψ is any such that the probabilities of propositional sets satisfy the usual rules, and for any set whose membership can be expressed as some (hyper-)arithmetical sentence P:

$$\mathbf{\Pr}[P] = \sup_{\aleph} \inf_{\beth} \mathbf{\Pr}[\mathrm{Game}(P, \aleph, \beth)]$$

THE REAL REASON THIS IS IMPORTANT

 "If a tree falls down in the forest, and no one's there to bet on it ... does it make a sound?"

Was an IDF Strike responsible for the AI-Ahli Hospital Explosion?

Israel • Israel-Hamas Conflict 2023 • Al-Ahli Hospital Explosion • Information war • Past Events



What caused the Late Bronze Age Collapse?

