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Towards a theoretical framework for algorithms of market dynamics

Abstract. The dynamics of markets outside of equilibrium have been studied in various algorithmic and game-theoretic contexts. If converging to an equilibrium, such dynamics can be interpreted as algorithms for equilibrium computation. I am interested in giving a behavioural foundation to out-of-equilibrium behaviour – and more generally, in rationalizing game dynamic or equilibrium computation algorithms as the optimal behaviour of locally optimising agents in the economy. To this end, I want to study Bayesian models of bounded rationality using imperfect information or rational inattention – as these models operate in the backdrop of one-value optimization, generalize them to multi-agent settings and apply them to the study of market dynamics. As this is fundamentally an interdisciplinary area, my work will have applications not only in the micro-foundation of out-of equilibrium behaviour, but also in computer science and machine learning.

Research background.

There has been a long-standing informal recognition of the fact that market dynamics is fundamentally a problem of information propagation. Information in the economy is generally dispersed and one of the key functions of a market is to aggregate this dispersed information with the help of prices. This is the view underlying Hayek's formulation of the economic calculation debate [1] later expanded upon in mechanism design [2].

While equilibrium dynamics are generally well understood, the process by which markets aggregate information and reach equilibrium is less understood even though several areas of economic theory study out-of-equilibrium behaviour: Dynamics of games have been extensively studied in the context of multi-agent learning [3] (wherein one seeks to describe the dynamics of a game as a strategy learning algorithm). Agent-based computational economics (ACE) [4] makes ad-hoc behavioral assumptions about agent behaviour to develop realistic models of out-of-equilibrium dynamics (such as in response to supply shocks [5–7]). However, these models do not attempt to *rationalize* game dynamics. While behavioral economist have tried to rationalise sub-optimal individual dynamic choice through imperfect information and rational inattention, e.g. [8, 9], this agenda has largely been confined to individual decision making instead of games and markets. I want to fill this gap, and work on the behavioral foundation of market dynamics focusing on the channel of incomplete information and its impact on individual and aggregate dynamic behaviour.

The general problem of studying market dynamics under the the lens of imperfect information of the agents is related to models of *information propagation* in networks. Most relevant to my interests, Bayesian belief propagation algorithms have been shown to be equivalent to simple agent-based models of bargaining in exchange networks [10–12], thereby founding belief propagation in precise ACE-like behavioural assumptions. However, like in ACE, these behavioral assumptions are ad-hoc, and not expressible in a general theoretical framework; their work cannot directly be extended to a different model of agent behavior, or to games other than bargaining games.

I believe that modelling game dynamics as the result of information propagation is a promising approach to rationalizing the former, and would be beneficial to the goal of finding a general theoretical framework for modeling market algorithms.

Research problems.

A basic "minimal working example" of a research problem for the above agenda is the study of message-passing algorithms such as Max-Sum as models for information propagation in markets. We refer to [13] for a rundown of the algorithm – at each iteration, an agent sends a message to its neighbours that encapsulates its own interests and the messages received from its own neighbours, and each agent chooses an optimal choice based on the information it has received.

I will use the algorithm to define a Bayesian game of message sending where agents are initially uninformed about other agents' preferences. Agents learn from message received over the course of the Max-Sum algorithm and use them to update their prior. As a first step I will investigate what the agents' priors should be such that the choices made at each iteration are Bayes-rational for each agent.

Message-passing algorithms are formulated in a non-strategic setting, and while we still assume this in our model (that the agents do not attempt to pick a choice in a way that optimally influences the future choices of other agents, but rather just to maximize some current game), formulating the game corresponding to a message-passing algorithm requires more detail on what the agents are permitted to do.

The second fundamental theorem of welfare economics [14] gives a correspondence between initial endowments and different efficient outcomes in a Walrasian setting; initial endowments are a special example of rights structures, as formalized mathematically in property rights theory e.g. [15, 16]. In my own previous work [16], I suggested that the second welfare theorem be generalized to a correspondence between rights structures and social welfare functions (i.e. that each rights structure maximizes a particular social welfare function at equilibrium). This is directly of interest to us, as we are interested in finding the rights structure implied by a particular message-passing algorithm (i.e. by a particular function being optimized).

Thus, I want to determine the precise correspondence between rights structures and the social welfare functions they maximize, and use this to formulate message-passing algorithms as equilibrium computation algorithms on games with rights structures.

Still, Max-Sum has many limitations, namely that it is computationally expensive to scale [17], which raises doubts as to how realistic it is as a model of economic computation in real markets. Thus, I intend to extend this work to other algorithms of market dynamics present in the literature. I briefly review other mechanisms that are particularly relevant for my research agenda, and highlight the differences between them and their limitations.

- *Algorithmic game theory* Equilibrium computation algorithms [18–20], in particular on graphs [21], double auctions [22–24], supply chain double-auctions [25, 26]. These algorithms are typically considered over the setting of an artificially-designed market, such as an online supply chain (rather than the natural dynamics of markets), and the dynamics of the game are thus "hard-coded" into the system, as it is an engineering problem.
- *Multi-agent learning* Repeated games, e.g. multi-agent reinforcement learning [27, 28], no-regret learning [29].

Applications and significance.

One of the main motivations for my research agenda is to give foundations for models of agent-based computational economics. A general theoretical framework in terms of information propagation will enable a wider and more systematic study of possible market dynamics than the ad-hoc assumptions currently seen in the literature.

In particular, my work will have implications in microfoundational research. Underlying microeconomic models have been proposed for a variety of observed macroeconomic phenomena, e.g. growth [30] and business cycles [31]. My proposed theoretical framework will bring greater scientific clarity to such phenomenological work, as it enables us to make precise scientific predictions, allowing the falsification of models of agent behavior and bounded rationality.

Furthermore, my investigation of right structures directly pertains to the theoretical foundations of welfare economics, and could in similar spirit pave the way for research into the effects of redistribution, or on the relative inequalities in preference fulfilment resulting from a particular rights structure.

Conversely, my work will also have implications for computer science and machine learning algorithms. The contrapositive of the quote "If your laptop can't find it, then neither can the market" (attributed to Kamal Jain by Christos Papadimitriou) is that a realistic model of information propagation in the market will allow for computationally efficient multi-agent co-ordination mechanisms. The development of a "natural" model of market dynamics from realistic assumptions about the temporal game will contribute directly to this goal. This serves to address the known computational inadequacies of existing message-passing algorithms like Max-Sum [17].

Furthermore, many classic machine learning/optimization algorithms may be mathematically equivalent to multi-agent co-ordination algorithms – this is shown by [10–12] for belief propagation, and I suspect that a differential version of Max-Sum may look a lot like backpropagation when implemented on acyclic graphs (there is a continuous version of Max-Sum in the literature [32], but it is not marginal in nature).While this line of research is more speculative, my research agenda in general may have implications for studying (or reverse-engineering) the convergence and complexity properties of traditional learning algorithms.

Tentative research plan.

In the proposal above, I've listed several closely-aligned problems that I am interested in. Below is a coarse tentative research plan/outline of the research part of my PhD:

Months 1-8. I will first work on the basic setting of Max-Sum without being too committed to a particular theoretical framework. I will work with the more basic setting of initial endowments to formulate the game of interest; based on the results and time constraints, I could then work on applications and analogies to classic optimization algorithms.

Months 9-16. Focusing on the aspects of right structure, I will generalize my insights from the previous phase. This has the potential to lead to several papers e.g. in conferences such as LOFT and EC, and journals like Games and Economic Behavior.

Months 17-24. Depending on the results of the previous phases, I will either focus on generalizing my work to algorithms other than message-passing/Max-Sum, or may decide to delve deeper into the applications of my research. In the latter case, I would focus primarily on proposing laws for the "natural dynamics" of markets.

Months 25-36. In the final year of my PhD, I will focus on exploring the applications I've previously described, as I believe my work will open many exciting new directions of future research beyond my PhD. The economic applications of my work are those that currently hold my interest, although this is subject to change based on what I discover in the process.

References

[1] F. A. von Hayek. "Economics and Knowledge". In: *Economica* 4.13 (1937), pp. 33–54. ISSN: 00130427, 14680335. URL: `http://www.jstor.org/stable/2548786`.

[2] F. Palda. *The Apprentice Economist: Seven Steps to Mastery*. Cooper-Wolfling, 2013. ISBN: 9780987788047. URL: `https://books.google.co.uk/books?id=bUBNAgAAQBAJ`.

[3] D. Fudenberg et al. *The Theory of Learning in Games*. Economics Learning and Social Evolution Series. MIT Press, 1998. ISBN: 9780262061940. URL: `https://books.google.co.uk/books?id=G6vTQFluxuEC`.

[4] William Brian Arthur. "Out-of-equilibrium economics and agent-based modeling". In: *Handbook of Computational Economics*. Vol. 2. Amsterdam: Elsevier, 2006. Chap. 32, pp. 1551–1564.

[5] Alexander Outkin et al. "An agent-based modeling approach to non-equilibrium dynamics of natural gas supply shock propagation". In: *International Association of Energy Economists 2014 Conference*. 2014.

[6] Mauro Napoletano, Jean-Luc Gaffard, and Zakaria Babutsidze. *Agent-based models: a new tool for economic and policy analysis*. Tech. rep. hal-01070338f. 2012.

[7] Dhruv Sharma. *Macroeconomic agent-based models: a statistical physics perspective*. Tech. rep. tel-03199888f. 2020.

[8] Xavier Gabaix. *Behavioral Inattention*. Working Paper 24096. National Bureau of Economic Research, Dec. 2017. DOI: `10.3386/w24096`. URL: `http://www.nber.org/papers/w24096`.

[9] Xavier Gabaix and David Laibson. *Myopia and Discounting*. Working Paper 23254. National Bureau of Economic Research, Mar. 2017. DOI: `10.3386/w23254`. URL: `http://www.nber.org/papers/laibson4`.

[10] Yashodhan Kanoria et al. *Bargaining dynamics in exchange networks*. 2011. arXiv: `1004.2079 [cs.GT]`.

[11] Yashodhan Kanoria et al. “Fast Convergence of Natural Bargaining Dynamics in Exchange Networks”. In: *Proceedings of the Twenty-Second Annual ACM-SIAM Symposium on Discrete Algorithms*. San Francisco, California: Society for Industrial and Applied Mathematics, 2011, pp. 1518–1537.

[12] Yashodhan Kanoria et al. *A Natural Dynamics for Bargaining on Exchange Networks*. 2010. arXiv: 0911.1767 [cs.GT].

[13] A. Rogers et al. “Bounded approximate decentralised coordination via the max-sum algorithm”. In: *Artificial Intelligence* 175.2 (2011), pp. 730–759. ISSN: 0004-3702. DOI: 10.1016/j.artint.2010.11.001.

[14] A. Mas-Colell, M.D. Whinston, and J.R. Green. “Equilibrium and its Basic Welfare Properties”. In: *Microeconomic theory*. Oxford student edition. Oxford University Press, 1995. Chap. 16. ISBN: 9780195102680. URL: https://books.google.co.uk/books?id=sQGDQgAACAAJ.

[15] Peter Gärdenfors. “Rights, games and social choice”. In: *Noûs* 15.3 (1981), pp. 341–356. ISSN: 00294624, 14680068. DOI: doi.org/10.2307/2215437.

[16] Abhimanyu Pallavi Sudhir. *A mathematical definition of property rights in a Debreu economy*. 2021. arXiv: 2107.09651 [econ.TH].

[17] Md. Mosaddek Khan, Long Tran-Thanh, and Nicholas R. Jennings. “A generic domain pruning technique for GDL-based DCOP algorithms in cooperative multi-agent systems”. In: *Proceedings of the 17th International Conference on Autonomous Agents and Multiagent Systems*. Stockholm, Sweden, 2018, pp. 1595–1603.

[18] Vijay V Vazirani. “Combinatorial algorithms for market equilibria”. In: *Algorithmic Game Theory*. New York: Cambridge University Press, 2007. Chap. 5, pp. 22–26, 103–134.

[19] Leigh Tesfatsion. *Agent-Based Computational Economics*. Working Paper 1. Iowa State University, 2003. URL: http://www2.econ.iastate.edu/tesfatsi/acewp1.pdf.

[20] Dhananjay K. Gode and Shyam Sunder. “Allocative Efficiency of Markets with Zero-Intelligence Traders: Market as a Partial Substitute for Individual Rationality”. In: *Journal of Political Economy* 101.1 (1993), pp. 119–137. DOI: 10.1086/261868.

[21] Sham M. Kakade, Michael Kearns, and Luis E. Ortiz. “Graphical Economics”. In: *Learning Theory*. Ed. by John Shawe-Taylor and Yoram Singer. Berlin, Heidelberg: Springer Berlin Heidelberg, 2004, pp. 17–32. ISBN: 978-3-540-27819-1.

[22] Rustam Tagiew. *Towards barter double auction as model for bilateral social cooperations*. 2009. arXiv: 0905.3709 [cs.GT].

[23] R.Preston McAfee. “A dominant strategy double auction”. In: *Journal of Economic Theory* 56.2 (1992), pp. 434–450. ISSN: 0022-0531. DOI: 10.1016/0022-0531(92)90091-U.

[24] Roger B Myerson and Mark A Satterthwaite. “Efficient mechanisms for bilateral trading”. In: *Journal of Economic Theory* 29.2 (1983), pp. 265–281. ISSN: 0022-0531. DOI: 10.1016/0022-0531(83)90048-0.

[25] M. Babaioff and N. Nisan. “Concurrent Auctions Across The Supply Chain”. In: *Journal of Artificial Intelligence Research* 21 (May 2004), pp. 595–629. ISSN: 1076-9757. DOI: 10.1613/jair.1316.

[26] Moshe Babaioff and William E. Walsh. “Incentive-compatible, budget-balanced, yet highly efficient auctions for supply chain formation”. In: *Decision Support Systems* 39.1 (2005). The Fourth ACM Conference on Electronic Commerce, pp. 123–149. ISSN: 0167-9236. DOI: 10.1016/j.dss.2004.08.008.

[27] Kaiqing Zhang, Zhuoran Yang, and Tamer Başar. *Multi-agent reinforcement learning: a selective overview of theories and algorithms*. 2021. arXiv: 1911.10635 [cs.LG].

[28] Lorenzo Canese et al. “Multi-agent reinforcement learning: a review of challenges and applications”. In: *Applied Sciences* 11.11 (2021). ISSN: 2076-3417. DOI: 10.3390/app11114948.

[29] Jan-P. Calliess and Geoffrey J. Gordon. "No-Regret learning and a mechanism for distributed multiagent planning". In: *Proceedings of the 7th International Joint Conference on Autonomous Agents and Multiagent Systems - Volume 1*. Estoril, Portugal: International Foundation for Autonomous Agents and Multiagent Systems, 2008, pp. 509–516. ISBN: 9780981738109.

[30] Gunnar Eliasson et al. *Microfoundations of Economic Growth: A Schumpeterian Perspective*. Selected papers from the International Schumpeter Society, Biennial Meeting (6 : 1996 : Stockholm). University of Michigan Press, 1998. ISBN: 9780472109043. URL: https://books.google.co.uk/books?id=ggfsAAAAMAAJ.

[31] Giovanni Dosi, Giorgio Fagiolo, and Andrea Roventini. "The microfoundations of business cycles: an evolutionary, multi-agent model". In: *Schumpeterian Perspectives on Innovation, Competition and Growth*. Springer, 2009, pp. 161–180.

[32] Thomas Voice et al. "A hybrid continuous Max-Sum algorithm for decentralised coordination". In: *19th European Conference on Artificial Intelligence*. 2010, pp. 61–66. URL: https://eprints.soton.ac.uk/271239/.